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Quantum dilaton supergravity in 2D with non-minimally coupled matter

Luzi Bergamin
Institute for Theoretical Physics,
Vienna University of Technology, 1040 Vienna, Austria,
email: bergamin@tph.tuwien.ac.at

Abstract

General $N = (1, 1)$ dilaton supergravity in two dimensions allows a background independent exact quantization of the geometric part, if these theories are formulated as specific graded Poisson-sigma models. In this work the extension of earlier results to models with non-minimally coupled matter is presented. In particular, the modifications of the constraint algebra due to non-minimal couplings are calculated and it is shown that quartic ghost-terms do not arise. Consequently the path-integral quantization as known from bosonic theories and supergravity with minimally coupled matter can be taken over.

Keywords: Supergravity models, 2D Gravity, BRST Quantization.

1 Introduction

The traditional formulation of $N = (1, 1)$ dilaton supergravity models in two dimensions is based upon superfields [11, 15]. Beside other problems, this approach has serious limitations concerning the quantum theory: Superspace techniques are known for the second order formulation with vanishing bosonic torsion, only. However, as is known from the bosonic case (refs. [9, 10] and refs. therein), a background independent quantization must start from a first order formulation, which in general works with non-vanishing bosonic torsion. This formulation can be interpreted as a Poisson-sigma model (PSM) [16, 17] of gravity where beside the zweibein also the spin connection appears as an *independent* variable together with new scalar fields ("target-space coordinates") on the 2D world sheet. Upon elimination of certain fields this formulation is locally and globally equivalent to the well-known second order formulation [13].

During the last years it has been shown that *graded* Poisson-sigma models (gPSMs) provide a natural extension to describe dilaton supergravity, if the target space is extended by a "dilatino" and the gauge fields comprise also the gravitino [7, 12]. Among the generic class of gPSM gravity models a certain subclass of "genuine" dilaton supergravity has been identified in [3, 4], which was shown to be equivalent to the model of ref. [15] when certain components of the superfield in the latter are properly expressed in terms of the fields appearing in the gPSM formulation. This equivalence also allowed to derive in a straightforward way minimal and non-minimal interactions with matter at the gPSM level [2].

All these considerations at the *classical* level provide the basis for a background independent quantization, analogous to the bosonic case. As shown in ref. [1], this program indeed can be taken over to the case of supergravity, although considerable computational complications arise. The result of [1] is restricted to minimally coupled matter and it is the purpose of the present work to extend it to non-minimal coupling.

This paper is organized as follows: section 2 briefly reviews the gPSM formulation of supergravity and the coupling to matter fields. In section 3 a Hamiltonian analysis is performed, whereby emphasis is placed on the difference to the case of minimal coupling. This is the necessary prerequisite for the construction of the BRST charge and the path integral quantization (sect. 4). Finally, appendix A summarizes our notations and conventions.

2 Graded Poisson-Sigma model and supergravity

A general gPSM consists of scalar fields $X^I(x)$, which are themselves ("target space") coordinates of a graded Poisson manifold with Poisson tensor $\{X^I, X^J\} = P^{IJ}(X) = (-1)^{IJ+1}P^{JI}(X)$. The index I, in the generic case, may include commuting as well as anti-commuting fields. In addition one introduces the gauge potential $A = \mathrm{d}X^I A_I = \mathrm{d}X^I A_{mI}(x)\,\mathrm{d}x^m$, a one form with respect to the Poisson structure as well as with respect to the 2D worldsheet. The gPSM action reads

$$\mathcal{S}_{gPSM} = \int_{\mathcal{M}} \mathrm{d}X^I \wedge A_I + \frac{1}{2}P^{IJ}A_J \wedge A_I \ . \tag{1}$$

As the Poisson tensor P^{IJ} must have vanishing Nijenhuis tensor the action (1) is invariant under the symmetry transformations $\delta X^I = P^{IJ}\varepsilon_J$, $\delta A_I = -\mathrm{d}\varepsilon_I - \left(\partial_I P^{JK}\right)\varepsilon_K\, A_J$, where the term $\mathrm{d}\epsilon_I$ in the latter relation provides the justification for calling A_I "gauge fields". If the Poisson tensor has a non-vanishing kernel—the actual situation in any application to 2D (super-)gravity due to the odd dimension of the bosonic part of the tensor—there exist (one or more) Casimir functions $C(X)$ obeying $\{X^I, C\} = 0$, which, when determined by the field equations, are constants of motion.

In the most immediate application to 2D supergravity the gauge potentials comprise the spin connection $\omega^a{}_b = \varepsilon^a{}_b\omega$, the dual basis e_a containing the zweibein and the gravitino ψ_α:

$$A_I = (A_\phi, A_a, A_\alpha) = (\omega, e_a, \psi_\alpha) \qquad\qquad X^I = (X^\phi, X^a, X^\alpha) = (\phi, X^a, \chi^\alpha) \tag{2}$$

The fermionic components ψ_α ("gravitino") and χ^α ("dilatino") for $N = (1,1)$ supergravity are Majorana spinors. The scalar field ϕ will be referred to as "dilaton". The remaining bosonic target space coordinates X^a correspond to directional derivatives of the dilaton in the second order formulation (cf. [3, 7]). Not any gPSM with this field content can be interpreted as supergravity. Local Lorentz invariance determines the ϕ-components of the Poisson tensor

$$P^{a\phi} = X^b\epsilon_b{}^a \ , \qquad\qquad P^{\alpha\phi} = -\frac{1}{2}\chi^\beta{\gamma_*}_\beta{}^\alpha \ , \tag{3}$$

and the supersymmetry transformation is encoded in $P^{\alpha\beta}$. Symmetry restrictions of the latter [3, 4] yields the gPSM *supergravity* class of theories (called "minimal field supergravity", MFS, in our present paper) with the Poisson tensor ($\chi^2 = \chi^\alpha\chi_\alpha$, $Y = X^aX_a/2$ and f' denotes $df/d\phi$)

$$P^{ab} = \left(V + YZ - \frac{1}{2}\chi^2\Big(\frac{VZ+V'}{2u} + \frac{2V^2}{u^3}\Big)\right)\epsilon^{ab} \ , \tag{4}$$

$$P^{\alpha b} = \frac{Z}{4}X^a(\chi\gamma_a\gamma^b\gamma_*)^\alpha + \frac{iV}{u}(\chi\gamma^b)^\alpha \ , \qquad P^{\alpha\beta} = -2iX^c\gamma_c^{\alpha\beta} + \big(u + \frac{Z}{8}\chi^2\big){\gamma_*}^{\alpha\beta} \ , \tag{5}$$

where the three functions V, Z and the "prepotential" u depend on the dilaton field ϕ only and must be related by $V(\phi) = -\frac{1}{8}\big((u^2)' + u^2 Z(\phi)\big)$. Inserting the Poisson tensor (3), (4) and (5) into equation (1) the ensuing action becomes (for simplicity the wedge symbols are omitted, D denotes the covariant derivatives $(De)_a = \mathrm{d}e_a + \omega\epsilon_a{}^b e_b$, $(D\psi)_\alpha = \mathrm{d}\psi_\alpha - \frac{1}{2}\omega{\gamma_*}_\alpha{}^\beta\psi_\beta$)

$$\begin{aligned}\mathcal{S}_{MFS} = \int_{\mathcal{M}} \Big(\phi\mathrm{d}\omega + X^aDe_a + \chi^\alpha D\psi_\alpha + \epsilon\Big(V + YZ - \frac{1}{2}\chi^2\Big(\frac{VZ+V'}{2u} + \frac{2V^2}{u^3}\Big)\Big)\\ + \frac{Z}{4}X^a(\chi\gamma_a\gamma^b e_b\gamma_*\psi) + \frac{iV}{u}(\chi\gamma^a e_a\psi) + iX^a(\psi\gamma_a\psi) - \frac{1}{2}\big(u + \frac{Z}{8}\chi^2\big)(\psi\gamma_*\psi)\Big) \ .\end{aligned} \tag{6}$$

The Poisson tensor has at least one (bosonic) Casimir function $C = e^Q\big(Y - u^2/8 + \chi^-\chi^+(u' + \frac{1}{2}uZ)/8\big)$. In certain situations additional (bosonic and fermionic) Casimir functions emerge [3, 7].

It has been proven in [3] that this class of supergravity models is equivalent to the superfield supergravity of Park and Strominger [15] upon elimination of auxilliary fields and a suitable redefinition of the gravitino. This equivalence can be used to derive from the superspace construction the matter coupling for MFS models, for details of the calculations we refer to [2]. A supersymmetric matter multiplet consists of a real scalar field f and a Majorana spinor λ_α. In case of non-minimal coupling a

coupling function $K(\phi)$ is introduced as well. After elimination of all auxiliary fields the matter action

$$\mathcal{S}_{(m)} = \int_{\mathcal{M}} \Big[K \Big(\frac{1}{2} \mathrm{d} f \wedge * \mathrm{d} f + \frac{i}{2} \lambda \gamma_a e^a \wedge * \mathrm{d} \lambda + i * (e_a \wedge * \mathrm{d} f) e_b \wedge * \psi \gamma^a \gamma^b \lambda + \frac{1}{4} * (e_b \wedge * \psi) \gamma^a \gamma^b e_a \wedge * \psi \lambda^2 \Big) + \frac{u}{8} K' \lambda^2 \epsilon - \frac{1}{4} K' (\chi \gamma_* \gamma^a \lambda) e_a \wedge * \mathrm{d} f - \frac{1}{32} \Big(K'' - \frac{1}{2} \frac{[K']^2}{K} \Big) \chi^2 \lambda^2 \epsilon \Big] \quad (7)$$

is found to be invariant under local non-linear supersymmetry transformations (cf. eqs. (2.12)-(2.17) and (6.23)-(6.27) of [2]).

3 Hamiltonian analysis

The primary goal of the present paper is to develop the systematics of the quantization of the action (6) together with matter couplings (7). The quantization is performed via a Hamiltonian analysis introducing Poisson brackets. The special case of minimal coupling ($K(\phi) \equiv 1$) was addressed in [1]. Here we concentrate onto the difficulties arising from a non-trivial coupling function K, for further details on the purely geometric models and minimal coupling refs. [1, 4] as well as the literature on non-supersymmetric models (ref. [10] and refs. therein) should be consulted.

In what follows quantities evaluated from the gPSM part of the action are indicated by (g), quantities from the matter extension by (m). This separation is possible as the matter action (7) does not contain derivatives acting on the MFS fields. Further all formulae are written in light-cone coordinates from know on.

In the geometrical sector we define the canonical variables and the first class primary constraints ($\approx$ means zero on the surface of constraints) from the Lagrangian $L_{(g)}$ in (1) ($\dot{q}^I = \partial_0 X^I$) by

$$X^I = q^I \ , \qquad \bar{q}^I = (-1)^{I+1} \frac{\partial L}{\partial \dot{\bar{p}}_I} \approx 0 \ , \qquad \frac{\partial L}{\partial \dot{q}^I} = p_I = A_{1I} \ , \qquad \bar{p}_I = A_{0I} \ . \quad (8)$$

From the Hamiltonian density ($\partial_1 = \partial$)

$$H_{(g)} = \dot{q}^I p_I - L_{(g)} = \partial q^I \bar{p}_I - P^{IJ} \bar{p}_J p_I \quad (9)$$

the graded canonical equations $\partial H_{(g)} / \partial p_I = (-1)^I \dot{q}^I$ and $\partial H_{(g)} / \partial q^I = -\dot{p}_I$ are consistent with the graded Poisson bracket for field monomials $\{q^I, p'_J\} = (-1)^I \delta^I_J \delta(x - x')$. The primes[1] indicate the dependence on primed world-sheet coordinates x, resp. x', x''. The Hamiltonian density (9) $H_{(g)} = G^I_{(g)} \bar{p}_I$ is expressed in terms of secondary constraints only:

$$\{\bar{q}^I, \int \mathrm{d}x^1 \ H_{(g)}\} = G^I_{(g)} = \partial q^I + P^{IJ} p_J \quad (10)$$

The extension to include conformal matter is straightforward. From the action (7) together with the matter fields $\mathfrak{q} = f$, $\mathfrak{q}^\alpha = \lambda^\alpha$ the canonical momenta[2]

$$\frac{\partial L_{(m)}}{\partial \dot{\mathfrak{q}}} = \mathfrak{p} = \frac{K}{e} \Big((p_{++} \bar{p}_{--} + p_{--} \bar{p}_{++}) \partial \mathfrak{q} - 2 p_{++} p_{--} \dot{\mathfrak{q}} + 2i \big(p_{++} p_{--} (\bar{p}_- \mathfrak{q}^- + \bar{p}_+ \mathfrak{q}^+) - \bar{p}_{++} p_{--} p_+ \mathfrak{q}^+ - p_{++} \bar{p}_{--} p_- \mathfrak{q}^- \big) \Big) + \frac{i}{2\sqrt{2}} K' (p_{++} q^+ \mathfrak{q}^+ + p_{--} q^- \mathfrak{q}^-) \ , \quad (11)$$

$$\frac{\partial L_{(m)}}{\partial \dot{\mathfrak{q}}^+} = \mathfrak{p}_+ = -\frac{K}{\sqrt{2}} p_{++} \mathfrak{q}^+ \ , \qquad \frac{\partial L_{(m)}}{\partial \dot{\mathfrak{q}}^-} = \mathfrak{p}_- = \frac{K}{\sqrt{2}} p_{--} \mathfrak{q}^- \quad (12)$$

are obtained. Analogous to (9) the Hamiltonian density from the matter Lagrangian in eq. (7) is defined as $H_{(m)} = \dot{\mathfrak{q}} \mathfrak{p} + \dot{\mathfrak{q}}^+ \mathfrak{p}_+ + \dot{\mathfrak{q}}^- \mathfrak{p}_- - L_{(m)}$, and the total Hamiltonian density is the sum of this contribution and (9). For the Poisson bracket of two matter field monomials one finds $\{\mathfrak{q}, \mathfrak{p}'\} = \delta(x - x')$

[1] Derivatives with respect to the dilaton are indicated by a prime as well. We leave this inconvenience of notation as dots are used to indicate derivatives with respect to x^0.

[2] In what follows, canonical variables are used throughout. $\sqrt{-g} = e = p_{--} \bar{p}_{++} - p_{++} \bar{p}_{--}$ in these variables.

and $\{\mathfrak{q}^\alpha, \mathfrak{p}'_\beta\} = -\delta^\alpha_\beta \delta(x-x')$. We do not provide the explicit form of the matter Hamiltonian, as it can again be written in terms of secondary constraints:

$$H = G^I \bar{p}_I \qquad G^I = G^I_{(g)} + G^I_{(m)} \qquad \{\bar{q}^I, \int dx^1\, H_{(m)}\} = G^I_{(m)} \tag{13}$$

The explicit expressions for the matter part of the secondary constraints read:

$$\begin{aligned} G^{++}_{(m)} &= -\frac{K}{4p_{++}}(\partial\mathfrak{q} - \frac{1}{K}\mathfrak{p})^2 + i(\partial\mathfrak{q} - \frac{1}{K}\mathfrak{p})(\frac{K}{p_{++}}p_+\mathfrak{q}^+ - \frac{K'}{4\sqrt{2}}\frac{p_{--}}{p_{++}}q^-\mathfrak{q}^-) + \frac{iK'}{4\sqrt{2}}(\partial\mathfrak{q} + \frac{1}{K}\mathfrak{p})q^+\mathfrak{q}^+ \\ &\quad + \frac{K}{\sqrt{2}}\mathfrak{q}^+\partial\mathfrak{q}^+ - \frac{K'}{2\sqrt{2}}\frac{p_{--}}{p_{++}}p_+q^-\mathfrak{q}^-\mathfrak{q}^+ - p_{--}\Big(\frac{uK'}{4} - \frac{1}{8}(K'' - \frac{K'^2}{K})q^-q^+\Big)\mathfrak{q}^-\mathfrak{q}^+ \end{aligned} \tag{14}$$

$$\begin{aligned} G^{--}_{(m)} &= \frac{K}{4p_{--}}(\partial\mathfrak{q} + \frac{1}{K}\mathfrak{p})^2 - i(\partial\mathfrak{q} + \frac{1}{K}\mathfrak{p})(\frac{K}{p_{--}}p_-\mathfrak{q}^- + \frac{K'}{4\sqrt{2}}\frac{p_{++}}{p_{--}}q^+\mathfrak{q}^+) + \frac{iK'}{4\sqrt{2}}(\partial\mathfrak{q} - \frac{1}{K}\mathfrak{p})q^-\mathfrak{q}^- \\ &\quad - \frac{K}{\sqrt{2}}\mathfrak{q}^-\partial\mathfrak{q}^- + \frac{K'}{2\sqrt{2}}\frac{p_{++}}{p_{--}}p_-q^+\mathfrak{q}^-\mathfrak{q}^+ + p_{++}\Big(\frac{uK'}{4} - \frac{1}{8}(K'' - \frac{K'^2}{K})q^-q^+\Big)\mathfrak{q}^-\mathfrak{q}^+ \end{aligned} \tag{15}$$

$$G^+_{(m)} = iK(\partial\mathfrak{q} - \frac{1}{K}\mathfrak{p})\mathfrak{q}^+ - \frac{K'}{2\sqrt{2}}p_{--}q^-\mathfrak{q}^-\mathfrak{q}^+ \tag{16}$$

$$G^-_{(m)} = -iK(\partial\mathfrak{q} + \frac{1}{K}\mathfrak{p})\mathfrak{q}^- + \frac{K'}{2\sqrt{2}}p_{++}q^+\mathfrak{q}^-\mathfrak{q}^+ \tag{17}$$

As the kinetic term of the matter fermion λ is first order only, this part of the action leads to constraints as well. From (12) the usual primary second-class constraints are deduced:

$$\Psi_+ = \mathfrak{p}_+ + \frac{K}{\sqrt{2}}p_{++}\mathfrak{q}^+ \approx 0 \qquad \Psi_- = \mathfrak{p}_- - \frac{K}{\sqrt{2}}p_{--}\mathfrak{q}^- \approx 0 \tag{18}$$

These second class constraints are treated by substituting the Poisson bracket by the "Dirac bracket" [6] $\{f,g\}^* = \{f,g\} - \{f,\Psi_\alpha\}C^{\alpha\beta}\{\Psi_\beta, g\}$, where $C^{\alpha\beta}C_{\beta\gamma} = \delta^\alpha_\gamma$ and $C_{\alpha\beta} = \{\Psi_\alpha, \Psi_\beta\}$. From (18) together with the definition of the canonical bracket the matrix $C_{\alpha\beta}$ follows as $C_{\alpha\beta} = \sqrt{2}K\text{diag}(-p_{++}, p_{--})$ The Dirac brackets among the fermionic matter variables are:

$$\{\mathfrak{q}^+, \mathfrak{q}'^+\}^* = \frac{1}{\sqrt{2}Kp_{++}}\delta(x-x') \qquad \{\mathfrak{q}^-, \mathfrak{q}^-\}^* = -\frac{1}{\sqrt{2}Kp_{--}}\delta(x-x') \tag{19}$$

$$\{\mathfrak{p}_+, \mathfrak{p}'_+\}^* = \frac{Kp_{++}}{2\sqrt{2}}\delta(x-x') \qquad \{\mathfrak{p}_-, \mathfrak{p}'_-\}^* = -\frac{Kp_{--}}{2\sqrt{2}}\delta(x-x') \tag{20}$$

$$\{\mathfrak{q}^+, \mathfrak{p}'_+\}^* = -\frac{1}{2}\delta(x-x') \qquad \{\mathfrak{q}^-, \mathfrak{p}'_-\}^* = -\frac{1}{2}\delta(x-x') \tag{21}$$

Moreover, q^{++}, q^{--} and p_ϕ have non-trivial Dirac brackets with the matter fermion:

$$\{q^{++}, \mathfrak{p}'_+\}^* = -\frac{K}{2\sqrt{2}}\mathfrak{q}^+\delta(x-x') \qquad \{q^{--}, \mathfrak{p}'_-\}^* = \frac{K}{2\sqrt{2}}\mathfrak{q}^-\delta(x-x') \tag{22}$$

$$\{q^{++}, \mathfrak{q}'^+\}^* = -\frac{\mathfrak{q}^+}{2p_{++}}\delta(x-x') \qquad \{q^{--}, \mathfrak{q}'^-\}^* = -\frac{\mathfrak{q}^-}{2p_{--}}\delta(x-x') \tag{23}$$

$$\{p_\phi, \mathfrak{p}'_+\}^* = \frac{K'}{2\sqrt{2}}p_{++}\mathfrak{q}^+\delta(x-x') \qquad \{p_\phi, \mathfrak{p}'_-\}^* = -\frac{K'}{2\sqrt{2}}p_{--}\mathfrak{q}^-\delta(x-x') \tag{24}$$

$$\{p_\phi, \mathfrak{q}'^+\}^* = \frac{K'}{2K}\mathfrak{q}^+\delta(x-x') \qquad \{p_\phi, \mathfrak{q}'^-\}^* = \frac{K'}{2K}\mathfrak{q}^-\delta(x-x') \tag{25}$$

All remaining brackets are unchanged.

An important step is the calculation of the algebra of secondary constraints

$$\{G^I, G'^J\}^* = G^K C_K{}^{IJ}\delta(x-x') \; . \tag{26}$$

For the matterless case the calculation has been performed in [4], the extension with minimally coupled matter in [1]. For all details of this part of the calculation we refer the reader to these two publications and simply state the result

$$C_{minK}{}^{IJ} = -\partial_K P^{IJ} \; , \tag{27}$$

where the index "min" stands for minimal coupling. For the structure functions of the full, non-minimally coupled theory the notation ${C_K}^{IJ} = {C_{minK}}^{IJ} + {\Delta C_K}^{IJ}$ is useful. Up to total derivatives[3] new contributions ${\Delta C_K}^{IJ}$ can then be summarized as follows:

$$\Delta {C_\phi}^{++|-} = p_{--}\frac{\partial}{\partial p_-}\Delta {C_\phi}^{++|--} \qquad \Delta {C_\phi}^{--|+} = -p_{++}\frac{\partial}{\partial p_+}\Delta {C_\phi}^{++|--} \tag{28}$$

$$\Delta {C_+}^{++|--} = -\frac{1}{2\sqrt{2}}\Delta {C_\phi}^{--|+} \qquad \Delta {C_-}^{++|--} = -\frac{1}{2\sqrt{2}}\Delta {C_\phi}^{++|-} \tag{29}$$

$$\Delta {C_\phi}^{+|-} = p_{++}p_{--}\frac{\partial}{\partial p_+}\frac{\partial}{\partial p_-}\Delta {C_\phi}^{++|--} \tag{30}$$

$$\Delta {C_+}^{++|-} = \frac{1}{2\sqrt{2}}\Delta {C_\phi}^{+|-} \qquad \Delta {C_-}^{--|+} = -\frac{1}{2\sqrt{2}}\Delta {C_\phi}^{+|-} \tag{31}$$

There remains the definition of $\Delta {C_\phi}^{++|--}$. From a straightforward but tedious calculation the result

$$\begin{aligned}\Delta {C_\phi}^{++|--} &= \frac{K'}{4p_{++}p_{--}}\Big((\partial\mathfrak{q} + \frac{1}{K}\mathfrak{p})(\partial\mathfrak{q} - \frac{1}{K}\mathfrak{p}) - 2i\big(p_-\mathfrak{q}^-(\partial\mathfrak{q} - \frac{1}{K}\mathfrak{p}) + p_+\mathfrak{q}^+(\partial\mathfrak{q} + \frac{1}{K}\mathfrak{p})\big)\Big) \\ &+ \frac{i}{8\sqrt{2}}\frac{K'^2}{K}\Big(\frac{q^-\mathfrak{q}^-}{p_{++}}(\partial\mathfrak{q} + \frac{1}{K}\mathfrak{p}) - \frac{q^+\mathfrak{q}^+}{p_{--}}(\partial\mathfrak{q} - \frac{1}{K}\mathfrak{p})\Big) + \frac{i}{4\sqrt{2}}(K'' - \frac{K'^2}{K})\Big(\frac{q^-\mathfrak{q}^-}{p_{++}}(\partial\mathfrak{q} - \frac{1}{K}\mathfrak{p}) - \frac{q^+\mathfrak{q}^+}{p_{--}}(\partial\mathfrak{q} + \frac{1}{K}\mathfrak{p})\Big) \\ &+ \frac{K'p_-p_+}{p_{--}p_{++}}\mathfrak{q}^-\mathfrak{q}^+ + \frac{1}{2\sqrt{2}}(K'' - \frac{K'^2}{K})(\frac{p_-\mathfrak{q}^+}{p_{--}} + \frac{p_+\mathfrak{q}^-}{p_{++}})\mathfrak{q}^-\mathfrak{q}^+ + (\partial_\phi - \frac{K'}{K})\Big(\frac{uK'}{4} - \frac{1}{8}(K'' - \frac{K'^2}{K})q^-q^+\Big)\mathfrak{q}^-\mathfrak{q}^+\end{aligned} \tag{32}$$

is obtained. One of the important open question concerns the interpretation of this result, to enable the application of the methods developed here in other theories, e.g. extended supergravity. We will comment on that point again below.

4 Quantization

As in the case of minimal coupling [1] two pairs of (anti-)ghosts are introduced for the primary constraints (b_I, p_b^I) and the secondary constraints (c_I, p_c^I), resp. The brackets among them are defined conveniently as

$$[b_I, p_b^J] = -(-1)^{(I+1)(J+1)}[p_b^J, b_I] = \delta_I^J\ , \qquad [c_I, p_c^J] = -(-1)^{(I+1)(J+1)}[p_c^J, c_I] = \delta_I^J\ . \tag{33}$$

To first order in homological perturbation theory the BRST charge Ω follows straightforwardly:

$$\Omega = \bar{q}^I b_I + G^I c_I - \frac{1}{2}(-1)^I p_c^K \partial_K P^{IJ} c_J c_I \tag{34}$$

In ref. [1] it was found that (34) is nilpotent for minimally coupled matter. The aim of this section is to check this result for the non-minimally coupled case. It is important to realize that the nilpotency of (34) is far from obvious in the present case: The symmetry of the theory is non-linear and moreover the matter extension is obtained from a superspace version by (non-trivial) eliminations of auxiliary fields. It is exactly this last point where quartic ghost terms usually appear in target-space supergravity.

Therefore it is inevitable to check the nilpotency of (34) carefully. We cannot present the lengthy and tedious calculation in all details, instead we make some comments on the most important steps and then present the final result.

1. Obviously all brackets with the first term in (34) vanish trivially, as $\bar{q}^I$ plays the rôle of a Lagrange multiplier in our approach. In fact, this part of the BRST charge simply could be omitted ("minimal phase space").

2. The remaining terms with zero anti-ghosts vanish by means of the constraint algebra (26), which is nothing than the definition of the BRST charge to first order in the homological perturbation theory. Nevertheless, there appear some subtleties with total derivatives, which are illustrated at hand of two examples:

[3] Of course the appearance of total derivatives suggests the possibility of central charge extensions of the algebra. Certainly this directions should be investigated, but this is not the aim of the present work.

(a) The total derivatives mentioned before eq. (28) turn into total derivatives including the ghosts and are again dropped. E.g. the bracket $\{G^+, G'^-\}^*$ generates the total derivative $\partial(\mathfrak{q}^+\mathfrak{q}^-)$, which in the present calculation turns into a total derivative $\partial(\mathfrak{q}^+c_+\mathfrak{q}^-c_-)$.

(b) A new subtlety appears as brackets of the form $\{G^{++}c_{++}, G'^{++}c'_{++}\}^*$ are symmetric and thus do not vanish trivially. Due to the statistics of the ghosts the only possible terms are proportional to $c_{++}\partial c_{++}$. However all terms of this kind are found to cancel.

3. The different contributions with one anti-ghost yield a lot of messy expressions. But at the end of the day one finds that—somehow miraculously—all contributions cancel. Again it is important to realize that contributions of the form $c_I\partial c_I$ for two anti-commuting ghosts must be checked as well. Notice that the ∂q^I part of the secondary constraints is important for these terms to cancel. As an example one finds that the combination $\partial q^{++} + K\mathfrak{q}^+\partial\mathfrak{q}^+/\sqrt{2}$ in G^{++} (cf. (14)) cancels all surface contributions from the commutation with $\mathfrak{q}^+$ according to (19) and (23). Also, the inverse powers of p_I are important to cancel contributions from the scalar matter fields.

4. Much simpler are contributions with two anti-ghosts. Most terms vanish identically as the structure functions from minimal coupling do not depend on the p_I. Again all contributions cancel.

Putting the pieces together we can conclude that the BRST charge (34) is nilpotent. Quartic ghost terms do not appear! In all steps of the calculation no simple symmetry principle can be found, which is responsible for this result. Rather an interplay of the structure of the secondary constraints, of the structure functions and of the Dirac brackets is relevant. Therefore we are not able to guess, whether this result will apply to a broader class of models (such as potentials with self-interaction or extended supergravity) or not.

As the Hamiltonian vanishes on the constraint surface it simply becomes $H_{gf} = \{\Omega, \Psi\}$ for some gauge fixing fermion Ψ. Adopting the result of [1] the multiplier gauge $\Psi = -ip_c^{++}$ is used[4]. This gauge-fixing entails the Eddington-Finkelstein form of the bosonic line element and the ensuing Lagrangian becomes

$$
\begin{aligned}
L_{gf} = {} & \dot{\bar{q}}^I \bar{p}_I + \dot{q}^I p_I + \dot{\mathfrak{q}}\mathfrak{p} + \dot{\mathfrak{q}}^\alpha \mathfrak{p}_\alpha + p_c^I \dot{c}_I + p_b^I \dot{b}_I - i\partial q^{++} - iP^{++|J} p_J - i(-1)^K p_c^I C_I{}^{++|K} c_K \\
& + \frac{i}{4}\frac{K}{p_{++}}(\partial\mathfrak{q} - \frac{1}{K}\mathfrak{p})^2 + (\partial\mathfrak{q} - \frac{1}{K}\mathfrak{p})(\frac{K}{p_{++}}p_+\mathfrak{q}^+ - \frac{K'}{4\sqrt{2}}\frac{p_{--}}{p_{++}}q^-\mathfrak{q}^-) + \frac{K'}{4\sqrt{2}}(\partial\mathfrak{q} + \frac{1}{K}\mathfrak{p})q^+\mathfrak{q}^+ \\
& - \frac{i}{\sqrt{2}}K\mathfrak{q}^+\partial\mathfrak{q}^+ + \frac{i}{2\sqrt{2}}K'\frac{p_{--}}{p_{++}}p_+q^-\mathfrak{q}^-\mathfrak{q}^+ + ip_{--}\Big(\frac{uK'}{4} - \frac{1}{8}(K'' - \frac{K'^2}{K})q^-q^+\Big)\mathfrak{q}^-\mathfrak{q}^+ \ .
\end{aligned} \tag{35}
$$

This important and surprising result shall now be used to formulate the quantum theory in terms of a path integral, as done for minimal coupling in [1] already (cf. also the literature on the bosonic case [10]). However, in a non-linear theory with structure *functions* rather than structure constants, the result of the Hamiltonian analysis must be used with care in a path integral formulation; there may appear ordering problems from the commutation of operators, which yield additional terms in the path integral where the variables are treated as classical fields. In bosonic gravity with non-minimal coupling [10] and in supergravity with minimal coupling [1] such ordering problems are absent. The important observation is that $[G^K, C_K{}^{IJ}] = 0$ without any ordering terms (cf. Appendix B.2 of [1]).

As in the simplified version with minimal coupling possible sources of ordering problems are the appearance of $\mathfrak{q}^+$ or $1/p_{++}$ together with q^{++}. However, it can be checked by explicit calculations that the new contributions from non-minimal coupling do not spoil this central relation. This behavior is possible as in the constraints G^{++} and G^+ $\mathfrak{q}^-$ and p_{--} appear in the combination $\mathfrak{q}^-p_{--}$ only, which is free of ordering terms in $[q^{--}, \mathfrak{q}^-p_{--}]$. Similar observations hold for $\Delta C_K{}^{IJ}$.

Therefore we can plug the result (35) into a generating functional of Green functions, which has to be integrated over all physical fields and all ghosts. Together with sources $\mathcal{J} = (j_{q^I}, j_p{}^I, J, J_\alpha)$ for the geometrical variables and for the matter fields resp. it reads:

$$
\mathcal{W}[\mathcal{J}] = \int \mathcal{D}(q^I, p_I, \bar{q}^I, \bar{p}_I, \mathfrak{q}, \mathfrak{p}, \mathfrak{q}^\alpha, \mathfrak{p}_\alpha, c_I, p_c^I, b_I, p_b^I) \exp\Big(i\int d^2x \, \big(L_{gf} + q^I j_{q^I} + j_p{}^I p_I + \mathfrak{q}J + \mathfrak{q}^\alpha J_\alpha\big)\Big) \tag{36}
$$

[4]Notice that according to our notation p_{++} is purely imaginary. Also, one component $(q^+, \mathfrak{q}^-)$ of a spinor is real, while the other $(q^-, \mathfrak{q}^+)$ is imaginary.

The gauge-fixed Hamiltonian, being independent of $\bar{q}^I$, $\bar{p}_I$, b_I and p_b^I, allows a trivial integration of all these fields. As the remaining ghosts appear at most bi-linearly in the action they can be integrated over, which leads to the super-determinant $\mathrm{sdet} M_I{}^J = \mathrm{sdet}\big(\delta_I{}^J \partial_0 + iC_I{}^{++|J}\big)$. The integration of $\mathfrak{p}_\alpha$ by means of the constraint (18) is trivial as well, while this is possible for the bosonic momentum $\mathfrak{p}$ of matter after a quadratic completion. This yields the effective matter Lagrangian

$$\begin{aligned} L_{(m)} = {} & \frac{i}{4Kp_{++}}\tilde{\mathfrak{p}}^2 + iKp_{++}\dot{\mathfrak{q}}^2 + \dot{\mathfrak{q}}\big(K\partial\mathfrak{q} - 2iKp_+\mathfrak{q}^+ + \frac{i}{2\sqrt{2}}K'(p_{--}q^-\mathfrak{q}^- + p_{++}q^+\mathfrak{q}^+)\big) + \frac{K'}{2\sqrt{2}}\partial\mathfrak{q}q^+\mathfrak{q}^+ \\ & - \frac{K}{\sqrt{2}}p_{++}\dot{\mathfrak{q}}^+\mathfrak{q}^+ + \frac{K}{\sqrt{2}}p_{--}\dot{\mathfrak{q}}^-\mathfrak{q}^- - \frac{i}{\sqrt{2}}K\mathfrak{q}^+\partial\mathfrak{q}^+ + ip_{--}\Big(\frac{uK'}{4} - \frac{1}{8}(K'' - \frac{1}{2}\frac{K'^2}{K})q^-q^+\Big)\mathfrak{q}^-\mathfrak{q}^+ \; . \end{aligned} \tag{37}$$

After integration over the shifted variable $\tilde{\mathfrak{p}}$ the complete Lagrangian is linear in the p_I, the determinant $\det Kp_{++}$ can be absorbed by a suitable redefinition of the path integral measure of $\mathfrak{q}$ and $\mathfrak{q}^\alpha$. Therefore, as in the bosonic case and in supergravity with minimal coupling, all geometric variables can be integrated out and one is left with the integration of the matter variables, which must be treated perturbatively. Although (37) shares many properties with the respective results from earlier calculations (bosonic gravity, supergravity with minimal coupling), an important difference should be mentioned: As a consequence of the elimination of (superspace) auxiliary fields the prepotential u appears in the matter Lagrangian (37) and thus plays the rôle of the prepotential of geometry as well as of a potential in the matter couplings. In all cases investigated so far, a strict separation of the potentials appearing in the geometric part (V, Z and u) and the one of the matter extension, K, occurred.

From the result (37), non-local vertices of the matter fields can be derived, which will be presented elsewhere. Another point to consider is the definition of the remaining path integral measures and matter-loop calculations. However, the model presented here is probably too complicated for explicit calculations as the question turns out to be very cumbersome in bosonic gravity with non-minimal coupling [14] and for supergravity with *minimal* coupling [1] already.

Another line of investigations is the generalization to extended supergravity [5] and/or to matter Lagrangians including self-interaction. Here a better understanding of the result of this work will be important. Indeed, the subtle interplay of the constraints (14)-(17), the structure functions (28)-(32) and the Dirac brackets (19)-(25) which led to the surprising result of this section was not foreseeable. However, to keep the computational complications manageable, exactly these important relations must be understood a priori. Indeed, the results obtained so far suggest that the first order formulation and the subsequent Hamiltonian quantization provide a powerful symmetry principle, similar to the one of superspace in the second order formulation. Nevertheless, in the case of matter couplings the basics of this principle are not yet understood sufficiently.

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A Notations and conventions

These conventions are identical to [7,8], where additional explanations can be found.

The summation convention is $NW \to SE$ and almost every bosonic expression is transformed trivially to the graded case when using this summation convention and replacing commuting indices by general ones. This is possible together with exterior derivatives acting *from the right*, only. Thus the graded Leibniz rule is given by $\mathrm{d}\,(AB) = A\mathrm{d}B + (-1)^B\,(\mathrm{d}A)B$.

In terms of anholonomic indices the metric is $\mathrm{diag}(1,-1)$ and the symplectic 2×2 tensors $\epsilon_{ab} = -\epsilon^{ab}$, $\epsilon_{\alpha\beta} = \epsilon^{\alpha\beta}$ are defined with $\epsilon_{01} = 1$. The metric in terms of holonomic indices is obtained by $g_{mn} = e_n^b e_m^a \eta_{ab}$ and for the determinant the standard expression $e = \det e_m^a = \sqrt{-\det g_{mn}}$ is used. The volume form reads $\epsilon = \frac{1}{2}\epsilon^{ab} e_b \wedge e_a$; by definition $*\epsilon = 1$.

The γ-matrices are used in a chiral representation:

$$\gamma^0{}_\alpha{}^\beta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \gamma^1{}_\alpha{}^\beta = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \gamma_*{}_\alpha{}^\beta = (\gamma^1\gamma^0)_\alpha{}^\beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{38}$$

Light-cone components are very convenient. As we work with spinors in a chiral representation we can use $\chi^\alpha = (\chi^+, \chi^-)$ and upper and lower chiral components are related by $\chi^+ = \chi_-$, $\chi^- = -\chi_+$, $\chi^2 = \chi^\alpha \chi_\alpha = 2\chi_- \chi_+$. Vectors in light-cone coordinates are given by $v^{++} = \frac{i}{\sqrt{2}}(v^0 + v^1)$, $v^{--} = \frac{-i}{\sqrt{2}}(v^0 - v^1)$. The additional factor i in permits a direct identification of the light-cone components with the components of the spin-tensor $v^{\alpha\beta} = \frac{i}{\sqrt{2}} v^c \gamma_c^{\alpha\beta}$. This implies that $\eta_{++|--} = 1$ and $\epsilon_{--|++} = -\epsilon_{++|--} = 1$. In light-cone coordinates each γ-matrix has exactly one non-vanishing entry, namely $(\gamma^{++})_+{}^- = \sqrt{2}i$, $(\gamma^{--})_-{}^+ = -\sqrt{2}i$.

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